NSWCCD-70-TR—2004/114 August 2004 Coated and Uncoated Models; What is the Difference? An Odyssey in Ten Parts

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Technical Report

Coated and Uncoated Models; What is the Difference? An Odyssey in Ten Parts

by

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ABSTRACT

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features that are encountered in a one-to-one correspondence, is, nonetheless, invaluable and most revealing in those features that are common in both. In this vein, the coming reports are offered.

INTRODUCTION

A report composed of several parts is presented. The report pertains to the analysis of the response of an externally driven complex dynamic system. Notwithstanding that there exists a number of all-purpose computer programs that attempt to achieve one-to-one correspondences with the response behavior of an actual dynamic system that is driven by sets of complex external sources, nonetheless, in contrast and by and large, in this report the analysis seeks merely a phenomenological correspondence. The report does not argue whether the one-to-one or the phenomenological correspondence is the superior or the inferior analysis; they both have a place in the sun and they are largely supplemental. The report does disclose an endeavor to account for some of the features in the responses of the complex that can be phenomenologically identified in the responses of simplified models that admit to simple analyses. A number of models are considered involving elaborations on a rudimentary model which is employed as an initial model. The aim is to decipher and identify a few of the mechanisms that influence the data derived off the fully fledged dynamic system. The utility of the modelings and analyses conducted in the report, although they are intended, as just mentioned, to achieve merely a phenomenological correspondence, may reveal ways and means by which maladies in the response of the complex may be beneficially controlled. A phenomenological correspondence of a mechanism is often sufficient to indicate which of the parameters may be altered to achieve a desired goal. Again, although the mechanism is not defined with the certainty of a one-to-one correspondence, at the very least, a well founded trend may be achieved by judiciously implementing these indicated parametric alterations.

In Part I a rudimentary model of the complex is constructed and analyzed. The curvatures in the complex are planed and the finitenesses of the complex are extended such that edges are eliminated. The physical model of the fully fledged dynamic system is depicted in Figure 01; as shown, this model is generic with an axis that runs along the x-axis. The generic model is finite and possesses both a cavity, in which the external sources are placed, and spatial discontinuities in the structural members. In this report, at best, these spatial discontinuities are simulated by placing regular ribs as a means of spatially extrapolating the structural members to achieve simpler analytical descriptions. The geometry and the coordinate system that is largely adapted in this effort are sketched in the model. A plane and infinite model, of the fully fledged dynamic system depicted in Figure 01, is correspondingly shown in Figure 02. This model is here designated the rudimentary model. The rudimentary model advantageously admits to an algebraic analysis. Utilizing this model, a number of phenomena are examined and accounted for. The underlying estimates are cast largely in terms of transfer functions. The transfer is from a component in the external sources in a fluid (fluid no. 2) on one side of the plane dynamic system into a radiated component in a fluid (fluid no. 1) on the other. The subsequent models in the various parts of the report are based largely on this rudimentary model. In particular, the algebraic nature of the analysis is jealously retained. Indeed, the basis for tying in with a rudimentary model stems from a desire to maintain the simplicity inherent in the analysis of this rudimentary model; notwithstanding that the rudimentary model is considered, in the first place, in order to set, in a simpler context, the parametric values and normalizations that the more elaborate models may employ. The rudimentary model may also serve to assess the sensitivities of these elaborations as this model is gradually upgraded. In this vein, any idiosyncrasies in the estimates of the transfer functions issued by the rudimentary model are accentuated and their true meanings are exposed. These idiosyncrasies may then be prevented from falsifying the explanations of data derived of both, full scale and scaled physical models, notwithstanding that some of the idiosyncrasies may be, thereby, authenticated. Thus, the rudimentary model is upgraded in Part II to accommodate the resonances and anti-resonances that are caused by a cavity, in Part III to accommodate resonances and anti-resonances that are caused by structural discontinuities and, finally, in Part IV both the cavity resonances and anti-resonances and the structural resonances and anti-resonances co-exist. Thus, in Part IV the interactions among these two sets of resonances and anti-resonances are accommodated. The influence of coating and damping on these resonances and anti-resonances are of special interest in these accommodations. With this interest in mind, in Parts V and VI the governing scaling laws, from full-scale structures to scaled structures, and vice versa are investigated. In Part VII implications of introducing coating also at the interface of the structure with the fluid (fluid no. 2) in which the external sources reside, are briefly accounted for. Finally, in Parts VIII, IX and X, curvatures and finitenesses, which are integral elements of the complex, are investigated. [cf. Figure 01 viz. Figure 02.] These investigations are carried out in a similar format to those pursued in Parts II, III, IV and VII.

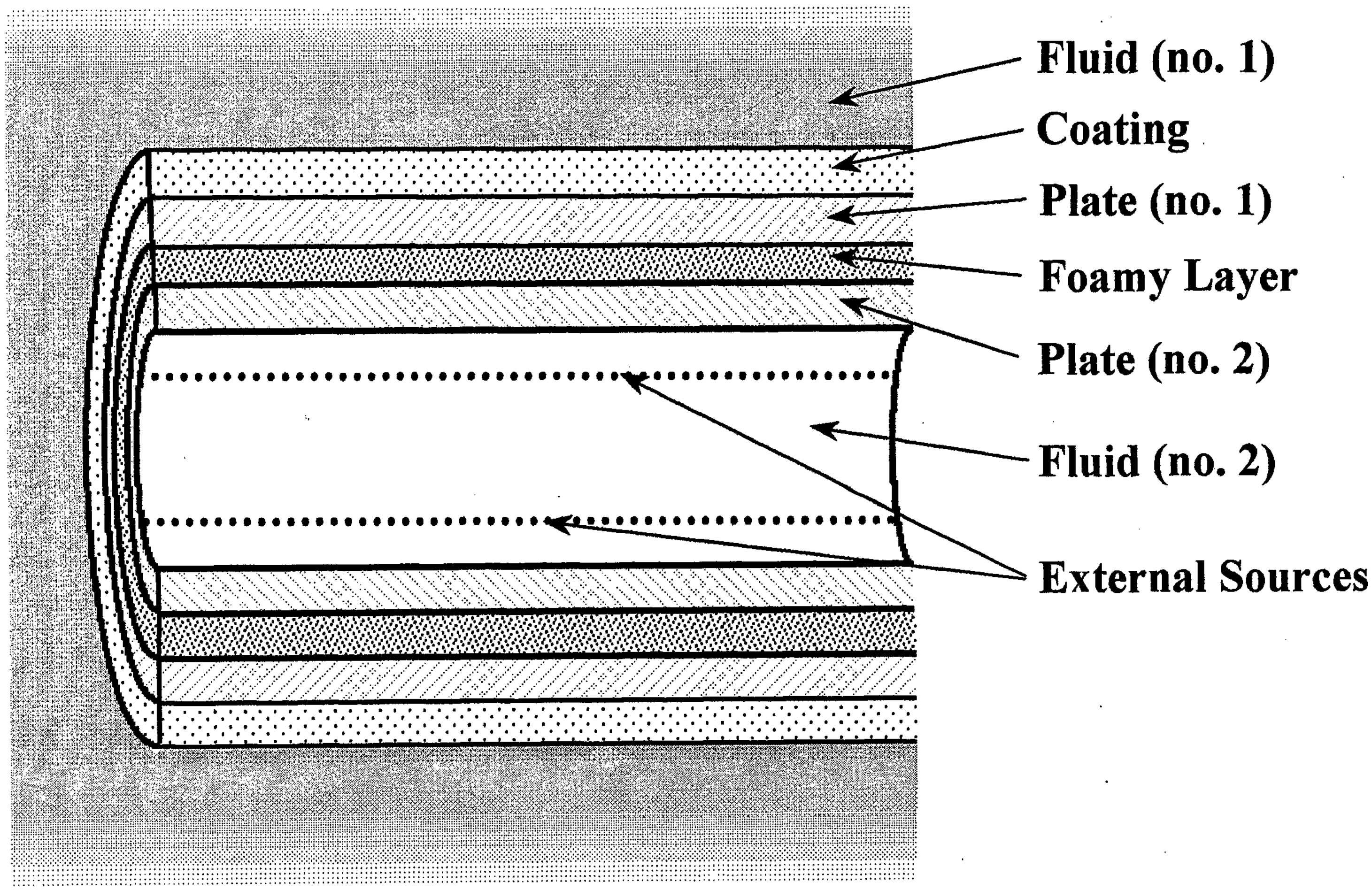


Figure O1. A sketch of a *fully fledged* complex dynamic system.

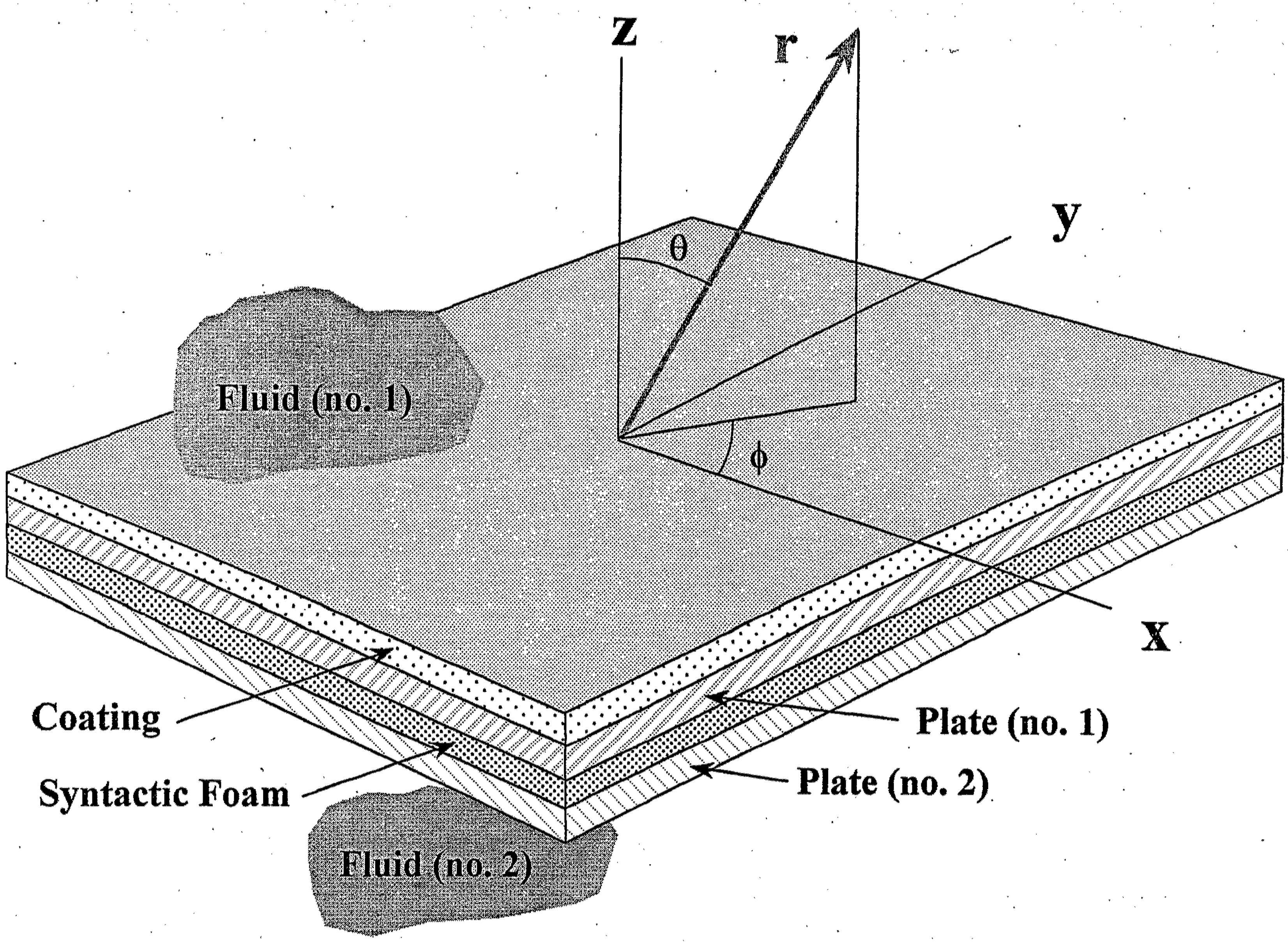


Figure O2. An equivalent rudimentary model of Figure O1 and coordinate system.

[cf. Figure O1.]

COATED AND UNCOATED MODELS; WHAT IS THE DIFFERENCE?

PART I: Rudimentary Model

G. Maidanik, K. J. Becker and L. J. Maga

Table of Contents to Part I

ABSTRACT TO PART I

A rudimentary model of a plane infinite structure is constructed. Fluids are placed on either side of this structure. In one fluid an external source is placed on a plane a short distance below the structure. These external sources generate acoustic radiation into the fluid atop the structure. The components that radiate are normalized by the corresponding components in the external sources. The so normalized radiating components are dubbed the transfer functions which vary as a function of the normalized frequency. The normalizing frequency is the resonance frequency of a harmonic oscillator comprising of the surface compliance of a coating and the surface mass of the plane structure. The coating, when present, is placed atop the surface of the structure and it then interfaces the top fluid. The ratio of the normalized transfer functions in the presence of coating to that in the absence of coating is depicted so that the influence of the coating on the transfer functions may be revealed. The model investigated herein supports a reasonable algebraic formalism that yields simple descriptions and admits to simple interpretations. A few examples are cited.

I1. Introduction.

A model is constructed and sketched in Figure I1. This model is a rudimentary model of the generic model depicted in Figure I2. An analysis is developed to investigate the transmission of sound to the far field resulting from external sources in the fluid (fluid no. 2) that interfaces a *bottom* plate (plate no. 2). The radiated sound is assessed in the fluid (fluid no. 1) occupying the space above a *top* plate (plate no. 1). The results of the analysis are reported in terms of transfer functions. The transfer functions are derived from the normalization of the radiating components by the corresponding components in the external source terms. The relevant components in the external source are those that generate the radiating components in the top fluid (fluid no. 1). The radiating components are assessed on the top plate. The corresponding external source componments are assessed on the plane in which they reside. This plane lies in the bottom fluid (fluid no. 2) a given distance below the bottom plate (plate no. 2). The ratio of the transfer functions with coating placed on the top plate (plate no. 1) to the corresponding transfer functions in the absence of coating are also presented. In this ratio the influence of coating the top plate (plate no. 1) on the transfer functions is revealed. Two models of the same ilk are examined. The first is depicted in Figure 1a and the second in Figure 1b. In the first, the top plate and the bottom plate (plate no. 1 and plate no. 2, respectively) are separated by a foamy layer. The foamy layer is assumed to introduce a compliant layer between the two plates. Any surface mass and surface stiffness that the foamy layer may have in addition to the layer of compliance, are analytically absorbed in the definition of the top and bottom plates. In the second model, which is depicted in Figure 1b, the two plates and the foamy layer between them are merged to form an *equivalent plate*. The velocities on the two outer surfaces of this plate are, thereby, specified to be equivalent to those in a thin plate. Analytically the two plates are merged by infinitely increasing the compliance of the foamy layer which effectively removes it. A more elaborate equivalent plate may be specified. However, such specification is not presented in this report.

The analyses that befit Figures 1a and 1b are reasonably elemental. To assist with these analyses the equivalent circuit diagrams depicted in Figures I3a and I3b are offered. In these figures the associated elemental formalisms are also presented. In Figure I3a, a three-by-three matrix is required whereas in Figure I3b a two-by-two matrix suffices. In order to employ the formalisms in computational tasks, the quantities and parameters in the formalisms are further detailed in subsequently stated equations; i.e., Equations (I1) - (I8) of Section I2. It is readily deduced, from the formalisms under consideration, that the *fluid-loadings* are damping terms. The damping is effective if the associated fluid is water and is ineffective if the associated fluid is air. [The ratio of the characteristic impedance of water to that of air is about (10^{-4}).] If the interface of the fluid with the plate is interrupted by a compliant coating in the way depicted in Figure I3, there is a potential for a resonance to occur between the surface mass of the plate, that now interfaces the coating, and the surface compliance of the coating. However, such a resonance may occur only if the fluid interfacing the coating is dense enough; e.g., is water. One may expect that if this resonance occurs it would show in the transfer function; does it?

I2. Parametric Definitions and Normalizations.

The surface impedances that are involved in the formalism of the various models, herein considered, are conveniently normalized by the absolute value of the total surface mass impedance of the structure that faces the two fluids on either side of it; i.e., the surface impedances are normalized by $|(i\omega m)| = (\omega m)$. The explicit expressions for the normalized surface impedances that cover those exhibited in Figures I1 and I3 are enumerated as follows: $Z_{f1}(k,\omega)$ is the normalized spectral surface impedance of the top fluid (fluid no. 1) in the plane of the structure

$$Z_{f1}(k,\omega) = [(\rho_1 c_1)/(\omega_1 m)](\omega_1/\omega)(\bar{k}_{31})^{-1} \quad , \text{(I1a)}$$

$Z_{f2}(k,\omega)$ is the normalized (spectral) surface impedance of the bottom fluid (fluid no. 2) in the plane of the structure

$$Z_{f2}(k,\omega) = [(\rho_2 c_2)/(\omega_2 m)](\omega_2/\omega)(\bar{k}_{32})^{-1} \quad , \text{(I1b)}$$

$Z_s(\omega)$ is the normalized (spectral) surface impedance of the foamy layer; this impedance is associated with the surface stiffness (compliance) of this layer

$$Z_s(\omega) = -i(\omega_s/\omega)^2(1+i\eta_s) \quad , (\text{I2a})$$

$Z_c(\omega)$ is the normalized (spectral) surface impedance of the coating; this impedance is associated with the surface stiffness (compliance) that the coating yields between the top fluid (fluid no. 1) and the surface of the top plate (plate no. 2)

$$Z_c(\omega) = -i(\omega_o/\omega)^2(1+i\eta_s) \quad , (\text{I2b})$$

$Z_1(k,\omega)$ is the normalized (spectral) surface impedance of the top plate (plate no. 1)

$$Z_1(k,\omega) = i(m_1/m)[1-i\eta_{m1} - \{k^4/(\omega\omega_1)^2\}(1+i\eta_1)] \quad , (\text{I3a})$$

and $Z_2(k,\omega)$ is the normalized (spectral) surface impedance of the bottom plate (plate no. 2)

$$Z_2(k,\omega) = i(m_2/m)[1-i\eta_{m2} - \{k^4/(\omega\omega_2)^2\}(1+i\eta_2)] \quad , (\text{I3b})$$

where (ω_1) and (ω_2) are the critical frequencies in the top plate (plate no. 1) and in the bottom plate (plate no. 2), respectively, (m_1) and (m_2) are the surface masses of the top plate (plate no. 1) and of the bottom plate (plate no. 2), respectively, $(\omega_s)^2$ defines the surface stiffness K_s of the foamy layer; i.e., $\omega_s^2 = (K_s/m)$ and $(\omega_o)^2$ defines the surface stiffness K_c of the coating; i.e., $\omega_o^2 = (K_c/m)$, the vectors $\{\rho_1, c_1\}$ and $\{\rho_2, c_2\}$ define the densities and the speeds of sound in the top fluid (fluid no. 1) and in the bottom fluid (fluid no. 2), respectively, and

$$\bar{k}_{3j} = [\{1-(kc_j/\omega)^2\}^{1/2} U\{1-kc_j/\omega)^2\} - i\{(kc_j/\omega)^2 - 1\}^{1/2} U\{(kc_j/\omega)^2 - 1\}] \quad . \text{(I4)}$$

Finally, η_s, η_c, η_1, η_2, η_{m1} and η_{m2} are the loss factors associated with the surface stiffness in the foamy layer, with the surface stiffness in the coating and with the surface stiffness and with the surface mass in the two plates, respectively. It is convenient and conducive to normalize (k) by (k_o), where $k_o = (\omega_o/c_1)$ and (ω_1), (ω_2) and (ω) by ω_o, where, $\omega_o = (K_c/m)^{1/2}$. With the understanding that (k) and (ω_1), (ω_2) and (ω) are so normalized , Equations (I2-3) stand as are, except that in Equation (I2b) $\omega_o \equiv 1$ and

$$Z_{f1}(k,\omega) = (\gamma_1)(\omega_1/\omega)(\bar{k}_{31})^{-1} \quad , \text{(I5a)}$$

$$Z_{f2}(k,\omega) = \gamma_1(\rho_2 c_2/\rho_1 c_1)(\omega_1/\omega)(\bar{k}_{32})^{-1} \quad , \text{(I5b)}$$

where $\gamma_1 = (\rho_1 c_1 / \omega_1 m)$ and the normalized wavenumbers $(\bar{k}_{31})$ and $(\bar{k}_{32})$ are the viable wavenumbers normal to the plane of the *structure* in the top fluid (fluid no. 1) and in the bottom fluid (fluid no. 2), respectively; namely

$$\bar{k}_{31} = [\{1-(k/\omega)^2\}^{1/2} U\{1-(k/\omega)^2\} - i\{(k/\omega)^2-1\}^{1/2} U\{(k/\omega)^2-1\}] \quad , \text{(I6a)}$$

$$\bar{k}_{32} = [\{1-(c_2/c_1)^2 (k/\omega)^2\}^{1/2} U\{1-(c_2/c_1)^2(k/\omega)^2\}$$
$$-i\{(c_2/c_1)^2 (k/\omega)^2 - 1\}^{1/2} U\{(c_2/c_1)^2 (k/\omega)^2 - 1\}] \quad . \text{(I6b)}$$

The equivalent plate is defined by the normalized surface impedance derived from Equations (I3a) and (I3b) in the form

$$(m_1/m)(1-i\eta_{m1}) + (m_2/m)(1-i\eta_{m2}) \Rightarrow (1-i\eta_m) \quad , \text{(I7a)}$$

$$(m_1/m)(\omega_1)^{-2}(1+i\eta_1) + (m_2/m)(\omega_2)^{-2}(1+i\eta_2) \Rightarrow (\omega_c)^{-2}(1+i\eta_p) \quad . \text{(I7b)}$$

Thus, the normalized surface impedance of the equivalent plate $Z_p(k,\omega)$ is stated in the form

$$Z_p(k,\omega) = i[(1-\eta_m) - \{k^4/(\omega\omega_c)^2\}(1+i\eta_p)] \quad , \text{(I8)}$$

where again, the wavenumber (k) and the frequencies (ω) and (ω_c) are normalized by k_o ; $k_o = (\omega_o / c_1)$ and by $\omega_o = (K/m)^{1/2}$, respectively. The normalized frequency (ω_c) is the critical frequency of the equivalent plate normalized, as are all frequencies, by (ω_o).

To render the computations explicit in the case when (ω_s) is finite; $\omega_s \cong \omega_o$ say, it is necessary to provide the explicit values for the parametric ratios: (m_1/m), (m_2/m), η_{m1}, η_{m2}, ω_1, ω_2, η_1, η_2, η_c, γ_1, (c_2/c_1) and (ρ_2/ρ_1). When (ω_s) is set infinite, which casts the plate-foamy layer-plate system in terms of an equivalent plate, it is then necessary to provide a smaller number of explicit values. The parametric ratios called for in this case are η_m, ω_c, η_p, η_c, γ_1, (c_2/c_1) and (ρ_2/ρ_1) only. Finally, if in addition it is imposed that the two fluids are identical, the number of explicit values is further reduced. The parametric ratios needed for the computations in this case are η_m, ω_c, η_p, η_c and γ_1.

I3. Standard Modeling.

Focusing attention on the model in Figure I1b, on the analysis presented in Figure I3b and on the explicit parametric definitions provided in Equations (I1) - (I8), a series of computations of the transfer functions is conducted and presented graphically in Figure I4. As already intimated the transfer function is defined in terms of the normalized pressure $\mathrm{Pf}(k,\omega)$ on the interface of the structure with one of the fluids. Were the fluid the top fluid (fluid no. 1), $\mathrm{Pf}(k,\omega) \Rightarrow \mathrm{Pf1}(k,\omega)$ and were the fluid the bottom fluid (fluid no. 2) $\mathrm{Pf}(k,\omega) \Rightarrow \mathrm{Pf2}(k,\omega)$ etc. It also follows that the coated transfer function $\mathrm{Tc}\,(k,\omega)$ may be expressed in the form

$$\mathrm{Pfi}\,(k,\omega) \begin{cases} \mathrm{Tci}\,(Z_{f1}, Z_c, Z_1, Z_s, Z_2, Z_{f2}) & ,\text{(I9a)} \\ \mathrm{Tci}\,(Z_{f1}, Z_c, Z_p, Z_{f2}) & ,\text{(I9b)} \end{cases}$$

and the uncoated transfer function $\mathrm{T}(k,\omega)$ may be similarly expressed in the form

$$\mathrm{Pfi}\,(k,\omega) \begin{cases} \mathrm{Ti}\,(Z_{f1}, |Z_c| \to \infty, Z_1, Z_s, Z_2, Z_{f2}) & ,\text{(I10a)} \\ \mathrm{Ti}\,(Z_{f1}, |Z_c| \to \infty, Z_p, Z_{f2}) & ,\text{(I10b)} \end{cases}$$

where $Z_{f1}, Z_c, Z_1, Z_2, Z_p, Z_s$ and Z_{f2} are defined in Figures I1 and I3 and in Equations (I1-I8). It further follows that

$$\mathrm{Pf1}\,(k,\omega) = Z_{f1}(k,\omega)\left[V_{f1}(k,\omega)/P_e(k,\omega)\right] \qquad \text{, (I11a)}$$

$$\mathrm{Pf2}(k,\omega) = \left\{1 - Z_{f2}(k,\omega)\left[V_2(k,\omega)/P_e(k,\omega)\right]\right\} \qquad \text{, (I11b)}$$

where V_{f1}, V_2 and P_e are also defined in Figures I1 and I3 and in Equations (I1-I8). With the use of Equation (I7) one may also state that

$$\mathrm{Tci}\,\left(Z_{f1}, Z_c, Z_1, \left|Z_s\right| \to \infty, Z_2, Z_{f2}\right) \Rightarrow \mathrm{Tci}\,\left(Z_{f1}, Z_c, Z_p, Z_{f2}\right) \qquad \text{, (I12a)}$$

$$\mathrm{Ti}\,\left(Z_{f1}, \left|Z_c\right| \to \infty, Z_1, \left|Z_s\right| \to \infty, Z_2, Z_{f2}\right) \Rightarrow \mathrm{Ti}\,\left(Z_{f1}, \left|Z_c\right| \to \infty, Z_p, Z_{f2}\right) \qquad \text{. (I12b)}$$

Finally and conventionally it is imposed herein and in other parts of this report that

$$\mathrm{Tc1} = \mathrm{Tc} \text{ and } \mathrm{T1} = \mathrm{T} \qquad \text{. (I13)}$$

In Figure I4a a typical transfer function under standard conditions, is presented. In the standard conditions, the fluids (fluid no. 1 and fluid no. 2) are identical and dense; e.g., water, the normalized critical frequency (ω_c) of the equivalent plate is chosen to be equal to ten (10), all loss factors are set equal to one-thousandth (10^{-3}), the coating, when present, has a surface compliance that with the surface mass of the equivalent plate defines the normalizing resonance frequency (ω_o) and the radiation to the far-field is beam-directed. The beam-directed transfer function is related to radiation that is directed normally to the plane in which the equivalent plate lies. The modeling is such that convolutions with respect to the normalized wavevector variable (k) are avoided. This avoidance allows the direction of the radiation relate the normalized wavevector variable (k) to the normalized frequency variable (ω); namely; $k \Rightarrow k(\omega)$. For example, the magnitude of (k) for a directed radiation to the far field is equal to $[\omega \sin(\theta)]$, where (θ) is the off normal direction. For beam-directed radiation $\theta \equiv 0$ and, therefore, $k \equiv 0$. The standard conditions are defined so that only exceptions to the standard conditions need to be specifically stated. The transfer function depicted in Figure I4a shows no resonance. In this figure the coating is absent and therefore the absence of the resonance, at and in the vicinity where the normalized frequency is equal to unity, is not surprising. In Figure I4a, at the lower range of the normalized frequency; i.e., $\omega \leq 1$, the surface fluid impedances exceed the surface (mass) impedance of the equivalent plate and the transfer function is half (1/2). The increase in the surface (mass) impedance of the equivalent plate with increase in the normalized frequency is largely responsible for the reduction in the transfer function as the normalized frequency increases so that the absolute value of the surface (mass) impedance exceeds that of the surface impedance of the fluids. It is noted that for the beam-directed radiation, which is the standard condition, the surface impedance of the equivalent plate is dominated by the surface mass. In Figure I4b a typical $(\pi/3)$ off beam-directed transfer function is depicted. For this direction the normalized wavevector $k = \{\omega \sin(\pi/3), 0\}$, where the normalizing wavenumber is (ω_o/c) with (c) being the sound speed in the fluids. [Hereafter, the azimuthal angle (φ) is set equal to zero; $\varphi \equiv 0$, so that (k) can be conveniently identified with (k_x), since $k_y \equiv 0$.] [cf. Figures I1a and I2.] As in Figure I4a, in Figure I4b the coating is absent. However, in this case the surface impedance of the equivalent plate transits from that of mass control, when $\omega \sin(\pi/3) < (\omega_c/\omega_o)$, to that of stiffness control, when $\omega \sin(\pi/3) > (\omega_c/\omega_o)$. At the normalized frequency of transition, where $\omega \sin(\pi/3) \cong (\omega_c/\omega_o)$, the surface impedance of the equivalent plate becomes negligible. Thus, at the normalized frequency of transition the transfer function assumes the value it has at low normalized frequency. In the normalized frequency of transition the surface impedance of the fluids again exceed that of the equivalent plate. The transition is clearly visible in Figure I4b. Beyond the normalized frequency of transition, the transfer function diminishes steeply with increase in the normalized frequency, since the increase in the surface impedance of the equivalent plate in the surface stiffness control region is $(\omega)^2$ steeper than that in the surface mass control region. Figure I4b is repeated in Figure I4c except that the loss factors in the plate are significantly increased as stated. [In this report the damping in a plate is defined in terms of two loss factors; one is a surface mass control and the second is a surface stiffness control. In this way the damping in a plate may be retained even for a beam-directed transfer function yielding, thereby, a more versatile modeling for the damping in a plate.]

I4. Standard Coating.

Now coating is introduced on the top surface of the equivalent plate. The top side of the coating interfaces then with the top fluid (fluid no. 1). [Again, one is reminded that the standard coating is one in which its surface compliance combines with the surface mass of the equivalent plate to yield the normalizing resonance frequency (ω_o).] Computations of the transfer function, corresponding to those in Figure I4a, are carried out with the standard coating present. The results are depicted in Figure I4d. Figure I4d shows no resonance behavior; notwithstanding that Figure I4d differs from Figure I4a. Indeed, to emphasize this difference Figure I4a is repeated in Figure I4e. It is observed that the only difference between Figures I4d and I4e is due to the presence of the coating in Figure I4d and the absence of the coating in Figure I4e. The influence of the coating is revealed in Figure I4f where the ratio of the transfer function depicted in Figure I4d to that depicted in Figure I4e, is shown. Clearly, the presence of the coating depresses the transfer function; starting and sustaining beyond the normalizing resonance frequency (ω_o), where $\omega > 1$.

I5. Assigning the Bottom Fluid as Air.

The analysis just developed suggests that a possible resonance, at and in the vicinity of $\omega \cong 1$, cannot occur unless the presence of the fluid (fluid no. 1) interfacing the equivalent plate atop is subdued by the surface compliance of the coating and, in addition, the fluid interfacing the bottom of the equivalent plate (plate no. 2) is also subdued. The subduction of this latter fluid (fluid no. 2) may be achieved, for example, by rendering the surface impedance of this fluid negligible; e.g., the fluid (fluid no. 2) is air, not water. The low characteristic impedance for this fluid (fluid no. 2) does result in a resonance between the coating and the equivalent plate when the fluid atop is neutralized by the coating. This is computationally demonstrated, for a typical transfer function, in Figure I5a. (Of course, this resonance is familiar to those who are engaged in providing anti-radiation coating for hulls.) In Figure I5b the corresponding transfer function in the absence of coating is depicted. The ratio of the transfer function in Figure I5a to that in Figure I5b is depicted in Figure I5c. The influence of the coating is, thus, revealed in Figure I5c. This figure demonstrates that the coating is an essential ingredient for the resonance at the lower frequency range and also for the large reduction in the transfer function at the higher frequency range. The peak in the resonance occurs when the normalized frequency is in the vicinity of unity; $\omega \cong 1$. [The frequency normalization, one recalls, is performed by the resonance frequency (ω_o) pertaining to the surface mass of the equivalent plate resonating with the surface compliance (inverse of the surface stiffness) of the coating. The higher frequency range commences an octave or so above that resonance frequency; $\omega \geq 2$ or 3.

I6. Special Transfer Functions.

Nonetheless, there is *a form* of a transfer function in which a resonance behavior similar to that depicted in Figure I5a exists, *even when the fluid interfacing the bottom of the equivalent plate is of a high characteristic impedance; e.g., that of water.* This resonance appears in the estimation of a *special* form of the transfer function in which the normalization *is not* performed in terms of the components in the external sources on a plane, but rather in terms of the corresponding components of the pressure field on the *interface* of the fluid (fluid no. 2) with the equivalent plate. This resonance is depicted in Figure I6c. What exactly is depicted in Figure 16c? In Figure 16a is depicted the normalized pressure field $\mathrm{Pf1}(k,\omega)$ on the top surface of the coating; on the interface of the coating with the top fluid (fluid no. 1). From Equations (I9-13) it follows that

$$\mathrm{Pf1}(k,\omega) = \mathrm{Tc}(k,\omega) = \mathrm{Tc}(\mathrm{Z}_{f1}, \mathrm{Z}_c, \mathrm{Z}_p, \mathrm{Z}_{f2}) \qquad .\text{(I14a)}$$

In Figure I6b is depicted the normalized pressure field $\mathrm{Pf2}(k,\omega)$ on the interface of the equivalent plate with the bottom fluid (fluid no. 2). From Equations (I9-I13) it follows that

$$\mathrm{Pf2}(k,\omega) = \mathrm{Tc2}(k,\omega) = \mathrm{Tc2}(\mathrm{Z}_{f1}, \mathrm{Z}_c, \mathrm{Z}_p, \mathrm{Z}_{f2}) \qquad .\text{(I14b)}$$

The transfer function $\mathrm{Tc2}(k,\omega)$ is as a legitimate (regular) transfer function as is $\mathrm{Tc}(k,\omega)$, stated in Equation (I14a). In both, in $\mathrm{Tc}(k,\omega)$ and in $\mathrm{Tc2}(k,\omega)$, the coating is in place. By definition, Figure I6c is derived from the ratio of the values in Figure I6a to those in Figure I6b. This ratio, from Equations (I9a and I9b) is

$$\mathrm{Pf1\_Pf2}(k,\omega) = [\mathrm{Pf1}(k,\omega)/\mathrm{Pf2}(k,\omega)] = [\mathrm{Tc}\,(k,\omega)/\mathrm{Tc2}\,(k,\omega)] \quad .(\mathrm{I14c})$$

The transfer function $\mathrm{Pf1\_Pf2}(k,\omega)$ is a *special* form of the transfer function in the presence of coating. In this special transfer function, a resonance is clearly present at and in the vicinity where the normalized frequency (ω) is equal to unity; i.e., $\omega = 1$. This resonance is, of course, absent in the corresponding regular transfer function $\mathrm{Pf1}(k,\omega) \equiv \mathrm{Tc}(k,\omega)$ depicted in Figure I6a. It can be shown that the particular normalization implied in the special transfer function, as stated in Equation (I14c), is tantamount to the removal of the surface impedance of the bottom fluid (fluid no. 2). Indeed, from Figures I1 and I3 and from Equations (I11a and I11b) one may derive that

$$\mathrm{Pf1\_Pf2}(k,\omega) = \mathrm{Tc}\,(Z_{f1}, Z_c, Z_p, |Z_{f2}| \to 0) \quad .(\mathrm{I14d})$$

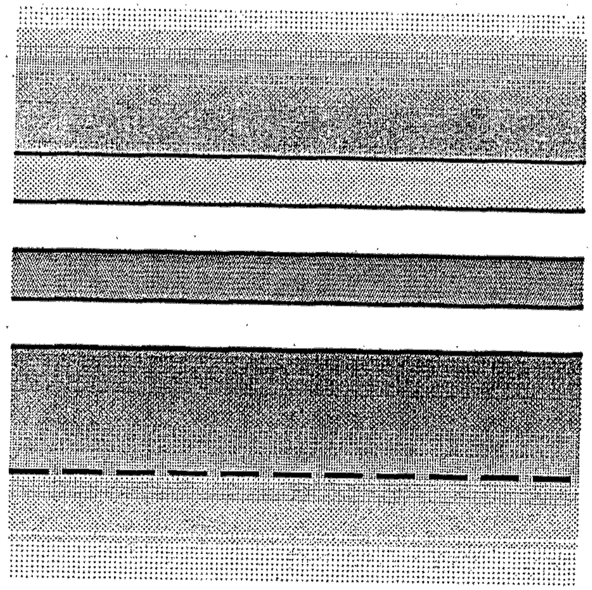
This case resembles then the situation cited in Section I5 and the proof is that Figure I5a closely allies with Figure I6c. In particular, both show the resonance that pertains to the surface compliance of the coating with the surface mass of the equivalent plate and in both the absolute value of the surface impedance of the bottom fluid (fluid no. 2) is rendered negligible. In the first the fluid is physically removed and in the second it is removed by a special choice for the normalization.

In Figures I6a, I6b and I6c the standard normalized wavenumber $k(\omega)$ is employed; this wavenumber is zero; i.e., $k(\omega) = 0$, indicating that a beam-directed radiation is being displayed. As in Section I2, it may be of interest to examine the special form of the transfer function for a $(\pi/3)$ off beam-directed radiation. Again, equating the normalized wavenumber $k(\omega)$ to $\omega \sin(\pi/3)$, Figures I6d, I6e and I6f are computed and shown. The anti-resonance and the resonance, respectively, in these figures at $\omega \sin(\pi/3) \cong (\omega_c/\omega_o)$ are associated with the transition of the surface impedance of the equivalent plate from mass control to stiffness control. This is reaffirmed in Figures I6g, I6h and I6i. These figures repeat Figures I6d, I6e and I6f, respectively, except that (ω_c) is set equal to five (5) instead of the standard value of ten (10). Consequently, in Figures I6g, I6h and I6i the transition from mass control to stiffness control occurs earlier in frequency than in Figures I6d, I6e and I6f, respectively. In the transition region the surface impedance of the equivalent plate is negligible and is damping control and is, therefore, dependent on the loss factors associated with the equivalent plate. The standard values of these loss factors is one-thousandth (10^{-3}). The dependence on the loss factors is made clear by examining Figures I6j, I6k and I6l. In these figures the loss factors of the equivalent plate are increased and are set equal to one-tenth (10^{-1}). The interpretation of Figures I6j, I6k and I6l, as compared with Figures I6g, I6h and I6i, is obvious enough. In this comparison the effect of the change in the loss factors cannot be missed.

Of course, special transfer functions may not only be defined to suite certain needs, but situations may arise in which one may attempt to use them for special diagnostic purposes. However, as they are defined, they are *special forms* for the transfer functions, they are not *regular* transfer functions. Here only regular transfer functions are in focus and to that matter only $\mathrm{Tc}(k,\omega)$ and $\mathrm{T}(k,\omega)$ are of paramount interest.

I7. Impaired Plate-Foamy Layer-Plate System.

Another way of subduing the surface impedance of the fluid (fluid no. 2) interfacing the bottom plate (plate no. 2) is by introducing a speculative surface compliance in the sandwich construction involving the top and bottom plates. [cf. Figures I1a and I3a where the foamy layer separates the top and bottom plates (plates no. 1 and no. 2, respectively.] This compliance can subdue the surface impedance of the bottom plate (plate no. 2) as well as the surface impedance of the fluid (fluid no. 2) that interfaces it. This phenomenon is typically demonstrated on a generic structure in Figure I7a. In this model the mechanism for rendering the surface impedances negligible is instituted by assigning the same surface compliance to the foamy layer as that assigned to the surface compliance of the coating, thereby, subduing the surface impedance of the top fluid (fluid no. 1), that of the bottom plate (plate no. 2) and that of the bottom fluid (fluid no. 2). This mechanism enables the coating placed on the top plate (plate no. 1) to resonate with the two compliances on either side of it. The resonance in this case occurs at $\omega \approx 2$, as observed in Figure I7a. In a corresponding transfer function, in which coating is absent, is depicted in Figure I7b. Again, the influence of the coating is exhibited in Figure I7c. In Figure I7c, the ratio of the transfer function in Figure I7a to the transfer function in Figure 7b, is presented. Again, the resonance is clearly visible with a peak at the resonance frequency caused by the interaction of the top plate (plate no. 1) with the surface compliances of the coating and that of the foamy layer. If the two plates are sandwiched properly as designed, this mechanism of introducing a resonance into the transfer function is moot. However, this mechanism must be kept in mind so that it may not become an inadvertent problem. The conversion of the plate-foamy layer-plate system into an equivalent plate is a major modeling step that requires caution.



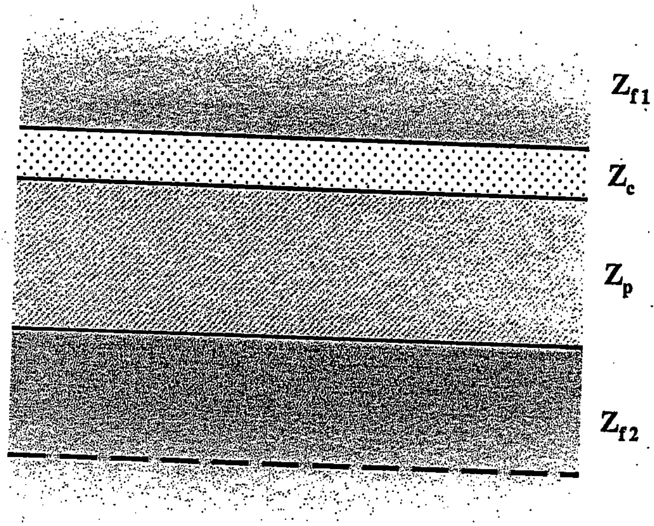
The surface impedances Z_{f1}, Z_c, Z_1, Z_s, Z_2 and Z_{f2} are normalized by the absolute value of the surface mass impedance (ωm), where (m) is the combined surface mass of the two plates (plate no. 1 and plate no. 2) and, therefore, the external drive (P_e) is also normalized by (ωm). In that sense (P_e) has the dimensionality of a velocity; e.g., the dimensionality of (V_{f1}), which is the velocity on the interface of the structure with fluid no. 1. The equation of motion is a three-by-three matrix:

$$\boldsymbol{ZV} = \boldsymbol{P}_e \;\; ; \;\; \boldsymbol{V} = \boldsymbol{YP}_e \qquad Z \equiv Z(k,\omega)$$

$$\boldsymbol{V} = \{V_{f1}, V_1, V_2\} \;\; ; \;\; \boldsymbol{P}_e = \{0, 0, P_e\} \qquad V \equiv V(k,\omega) \qquad P_e \equiv P_e(k,\omega)$$

where (k), which is notated as a scalar is in fact a wavevector in the plane of the system $\boldsymbol{k} \equiv \{k_x, k_y\}$ and (ω) is the frequency variable.

Figure I1a. Fluid (no. 1) - coating - plate (no. 1) - foamy layer - plate (no. 2) - fluid (no. 2) plane dynamic system that is externally driven by sources that are confined on a parallel plane and reside in the fluid (fluid no. 2) below.



The surface impedances Z_{f1}, Z_c, Z_p and Z_{f2} are normalized by the absolute value of the surface mass impedance (ωm), where (m) is the surface mass of the equivalent plate (plate no. 2) and, therefore, the external drive (P_e) is also normalized by (ωm). In that sense (P_e) has the dimensionality of a velocity; e.g., the dimensionality of (V_{f1}), which is the velocity on the interface of the structure with fluid no. 1. The equation of motion is a two-by-two matrix:

$$\boldsymbol{Z V} = \boldsymbol{P}_e \ ; \ \boldsymbol{V} = \boldsymbol{Y P}_e \qquad Z \equiv Z(k,\omega)$$

$$\boldsymbol{V} = \{V_{f1}, V_p\} \ ; \ \boldsymbol{P}_e = \{O, P_e\} \qquad V \equiv V(k,\omega)$$

$$P_e \equiv P_e(k,\omega)$$

where (k), which is notated as a scalar is in fact a wavevector in the plane of the system $\boldsymbol{k} \equiv \{k_x, k_y\}$ and (ω) is the frequency variable.

Figure I1b. The same as I1a except that the plate (no. 1) - foamy layer - plate (no. 2) portion of the plane dynamic system is merged to form an equivalent plate.

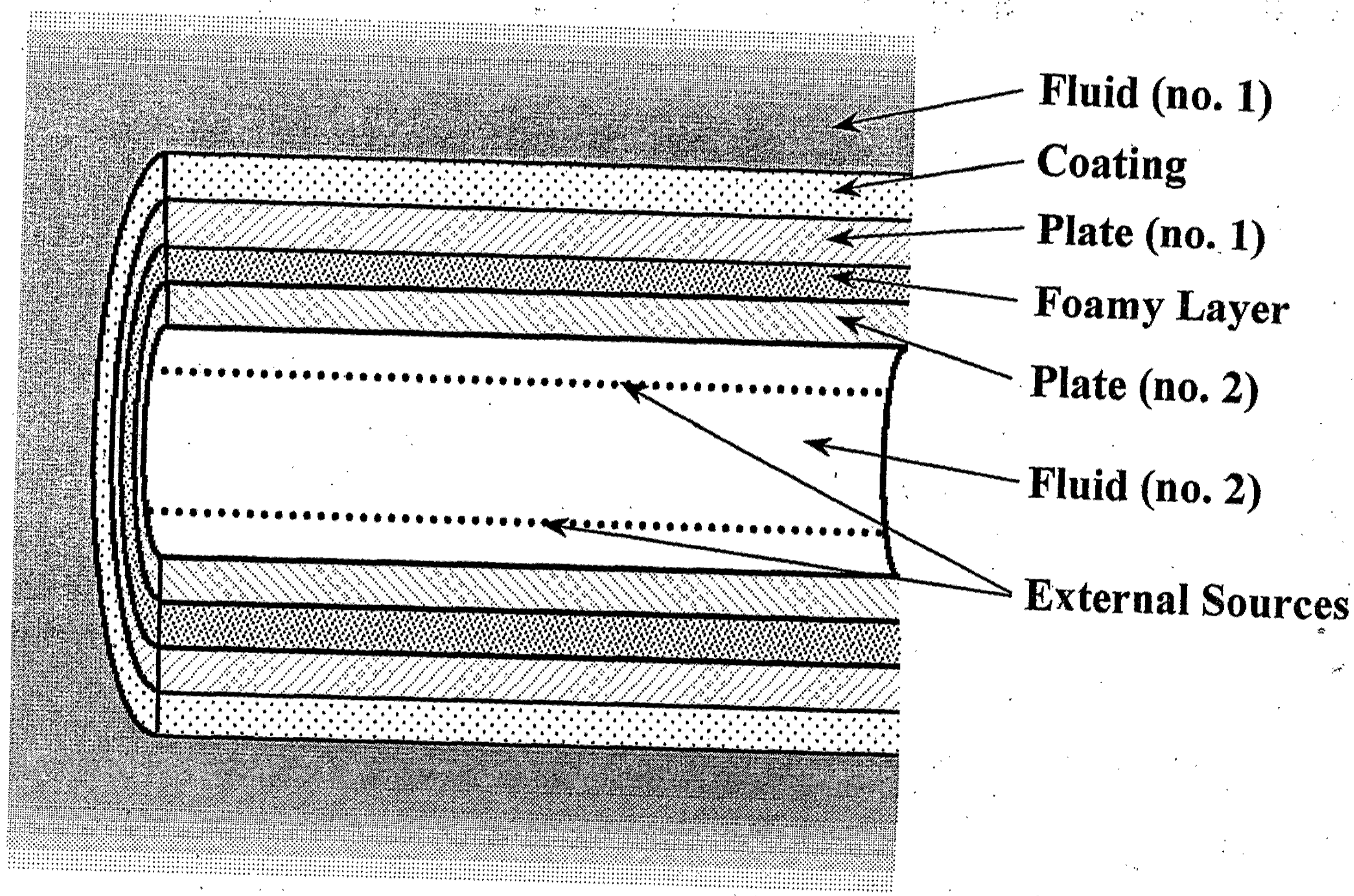
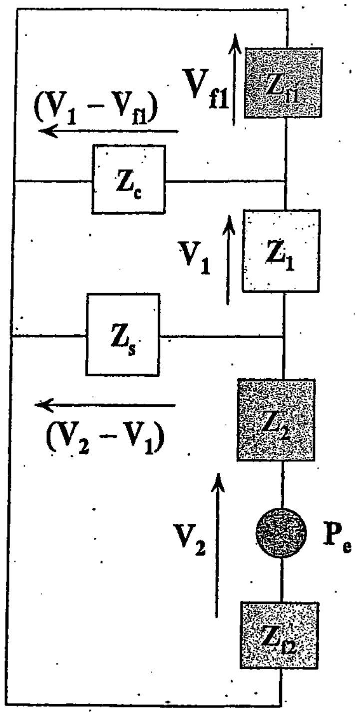


Figure I2. A generic model of a complex dynamic system. [cf. Figures O1 and O2.]



$$\boldsymbol{Z}\boldsymbol{V} = \boldsymbol{P}_e \;;\; \boldsymbol{V} = \boldsymbol{Y}\boldsymbol{P}_e$$

$$\boldsymbol{V} = \{V_{f1}, V_1, V_2\} \quad ; \quad \boldsymbol{P}_e = \{0, 0, P_e\}$$

$$\boldsymbol{Z} = \begin{matrix} (Z_{f1}+Z_1+Z_c) & -Z_c & 0 \\ -Z_c & (Z_s+Z_1+Z_c) & -Z_s \\ 0 & -Z_s & (Z_s+Z_2+Z_{f2}) \end{matrix}$$

$$\boldsymbol{Y} = \begin{matrix} Y_{f1f1} & Y_{f11} & Y_{f12} \\ Y_{1f1} & Y_{11} & Y_{12} \\ Y_{2f1} & Y_{21} & Y_{22} \end{matrix}$$

$$Y_{f1f1} = |\boldsymbol{Z}|^{-1}(Z_c+Z_1+Z_s)\{Z_2+Z_{f2}+\overline{Z_s(Z_c+Z_1)}\} \;;\; Y_{f11} = Y_{1f1} = |\boldsymbol{Z}|^{-1} Z_c(Z_s+Z_2+Z_{f2}) \;;$$

$$Y_{11} = |\boldsymbol{Z}|^{-1}(Z_{f1}+Z_c)(Z_s+Z_2+Z_{f2}) \;;\; Y_{12} = Y_{21} = |\boldsymbol{Z}|^{-1}(Z_{f1}+Z_c)\,Z_s \;;$$

$$Y_{22} = |\boldsymbol{Z}|^{-1}(Z_{f1}+Z_c)\{Z_s+Z_1+\overline{Z_{f1}Z_c}\} \;;\; Y_{2f1} = Y_{f12} = |\boldsymbol{Z}|^{-1} Z_c Z_s \;;$$

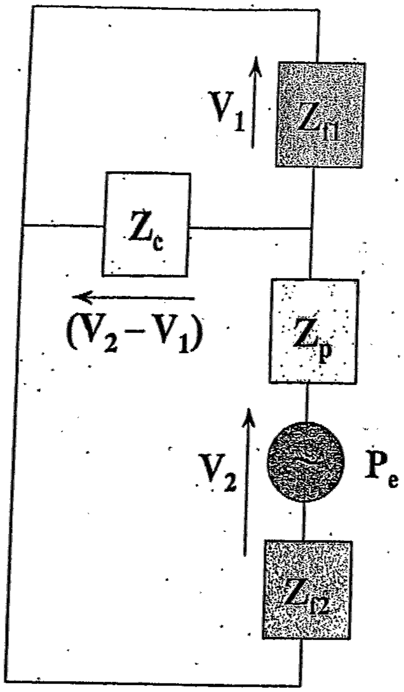
$$|\boldsymbol{Z}| = (Z_{f1}+Z_c)(Z_s+Z_2+Z_{f2})\{\overline{Z_cZ_{f1}} + Z_1 + \overline{Z_s(Z_2+Z_{f2})}\}$$

where

$$\overline{Z_{f1}Z_c} = Z_{f1}Z_c(Z_c+Z_{f1})^{-1} \quad ; \quad \overline{Z_s(Z_2+Z_{f2})} = Z_s(Z_2+Z_{f2})(Z_s+Z_2+Z_{f2})^{-1}$$

Figure I3a. The equivalent electrical circuit diagram representing the model depicted in Figure I1a and the formalism that relates the response vector $(\boldsymbol{V})$ to the external drive vector $(\boldsymbol{P}_e)$. That relationship is governed by the normalized impedance matrix $(\boldsymbol{Z})$.

$$ZV = P_e \; ; \; V = YP_e$$



$$V = \{V_{f1}, V_2\} \quad ; \quad P_e = \{0, P_e\}$$

$$Z = \begin{matrix} (Z_c + Z_{f1}) & -Z_c \\ -Z_c & (Z_{f2} + Z_p + Z_c) \end{matrix}$$

$$Y = \begin{matrix} Y_{f1f1} & Y_{f12} \\ Y_{2f1} & Y_{22} \end{matrix}$$

$$Y_{f1f1} = |Z|^{-1}(Z_{f2} + Z_p + Z_c) \; ; \; Y_{22} = |Z|^{-1}(Z_c + Z_{f1})$$

$$Y_{2f1} = Y_{f12} = |Z|^{-1} Z_c \; ; \; |Z| = (Z_c + Z_{f1})(Z_{f2} + Z_p + \overline{Z_{f1}Z_c})$$

where

$$\overline{Z_{f1}Z_c} = Z_{f1}Z_c(Z_c + Z_{f1})^{-1}$$

Figure I3b. The equivalent electrical circuit diagram representing the model depicted in Figure I1b and the formalism that relates the response vector (V) to the external drive vector (P_e). That relationship is governed by the normalized impedance matrix (Z).

Figure I4a. The uncoated transfer function, as a function of the normalized frequency, under standard conditions.

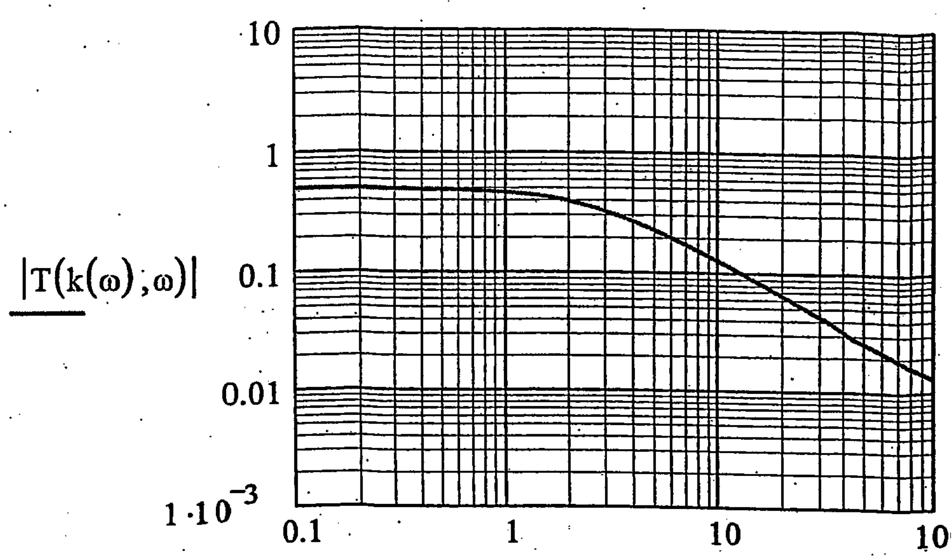


Figure I4b. The transfer function, as a function of the normalized frequency, under standard conditions except that the radiation is assessed $(\pi/3)$ off beam-direction; $k = \omega \sin(\pi/3)$.

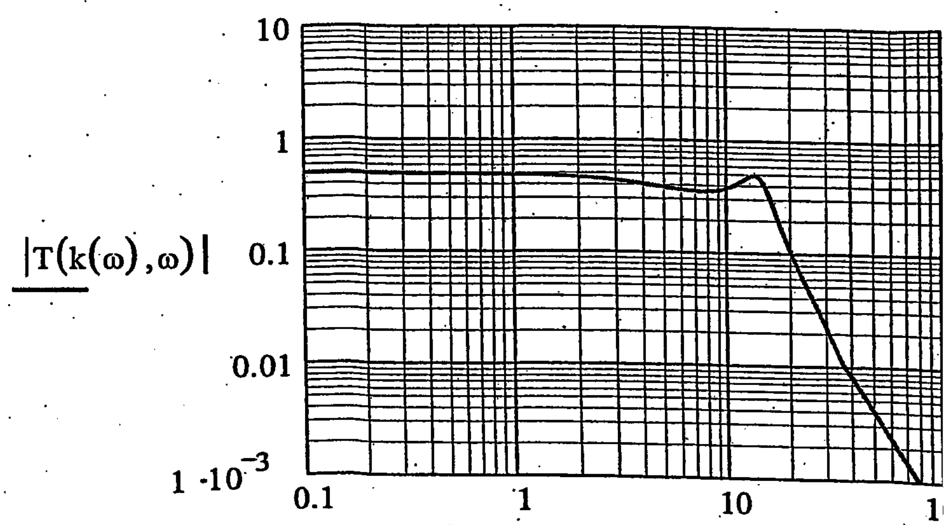


Figure I4c. Figure I4b is repeated except that the loss factors in the equivalent plate are significantly increased; in this figure $\eta_m = \eta_p = 10^{-1}$.

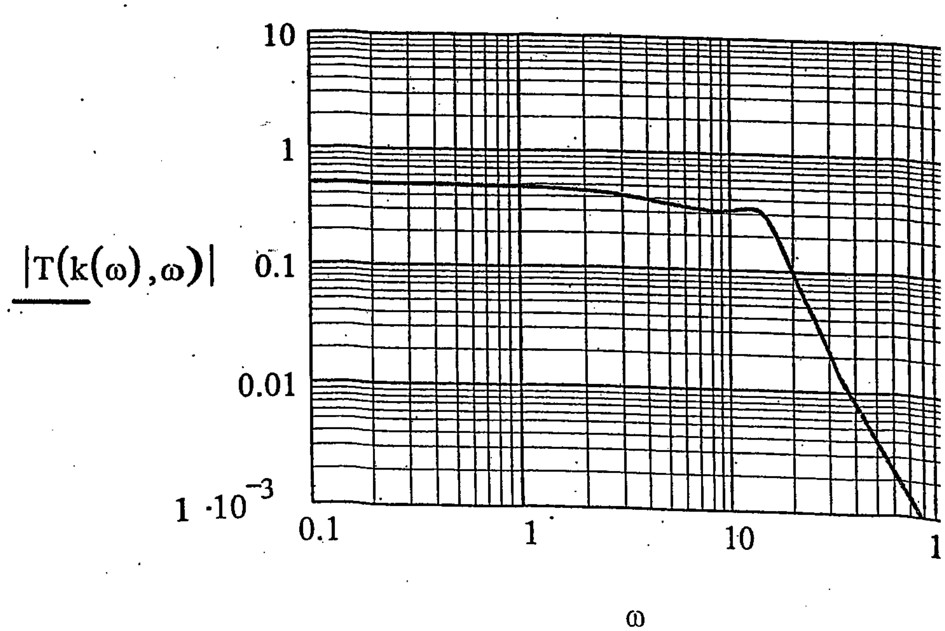


Figure I4d. The coated transfer function, as a function of the normalized frequency, under standard conditions. [cf. Figure I4a.]

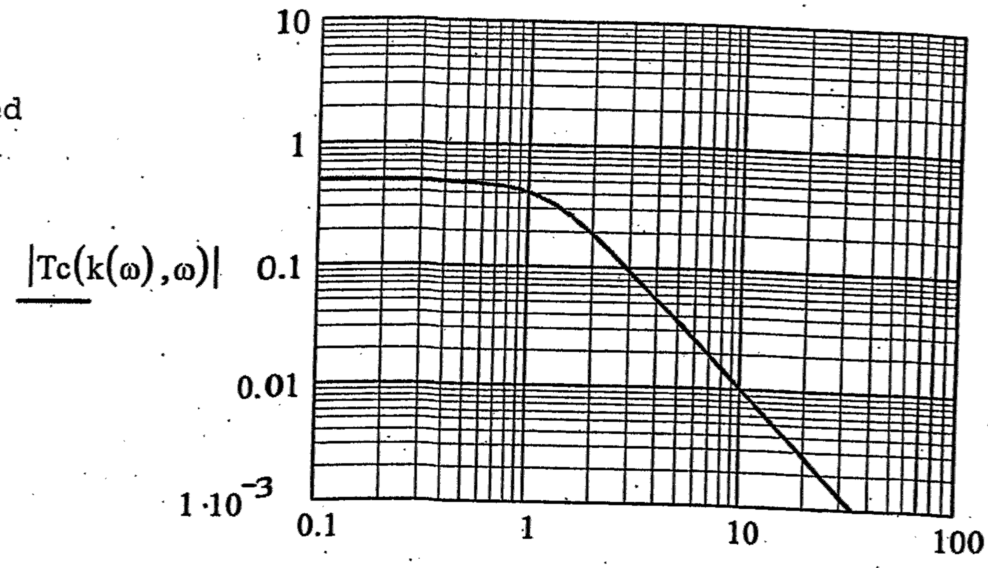


Figure I4e. Figure I4a is repeated.

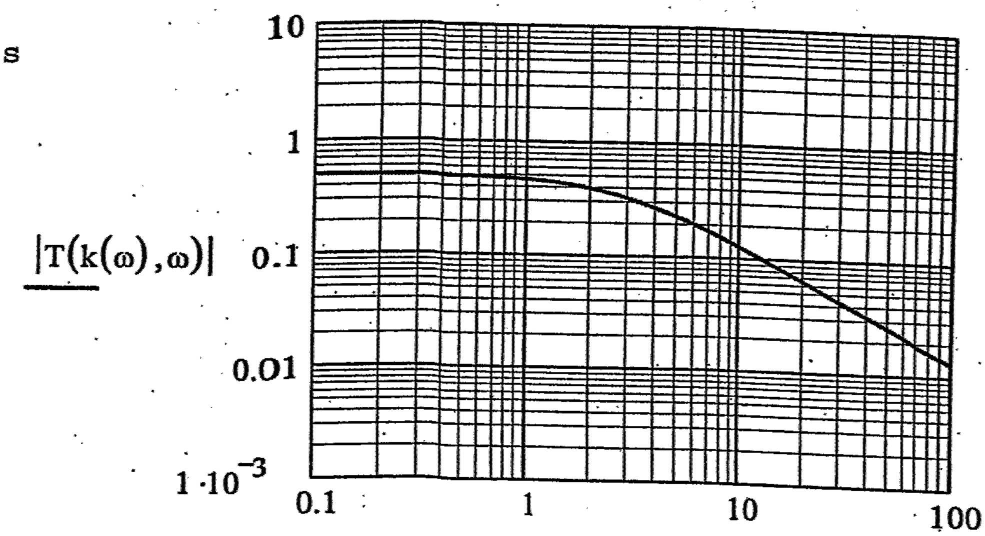


Figure I4f. The ratio of the transfer functions, as a function of the normalized frequency. The ratio is of the transfer function depicted in Figure I4d to that depicted in Figure I4e, thereby, revealing the influence of the coating on the transfer function.

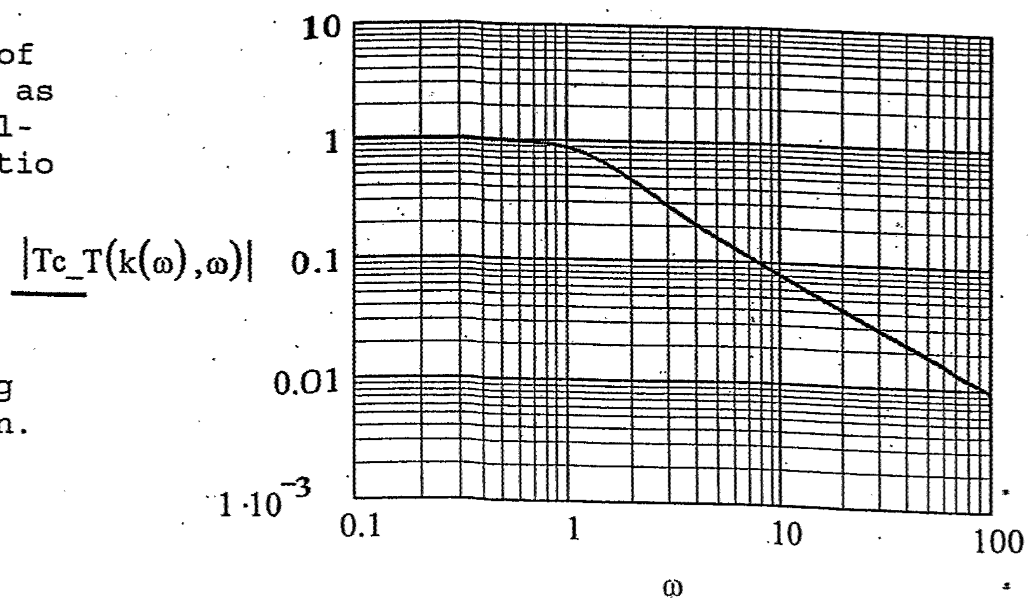


Figure I5a. The coated transfer function, as a function of the normalized frequency, under standard conditions except that the normalized surface impedance of the bottom fluid (fluid no. 2) is reduced by a factor of (10^{-4}).

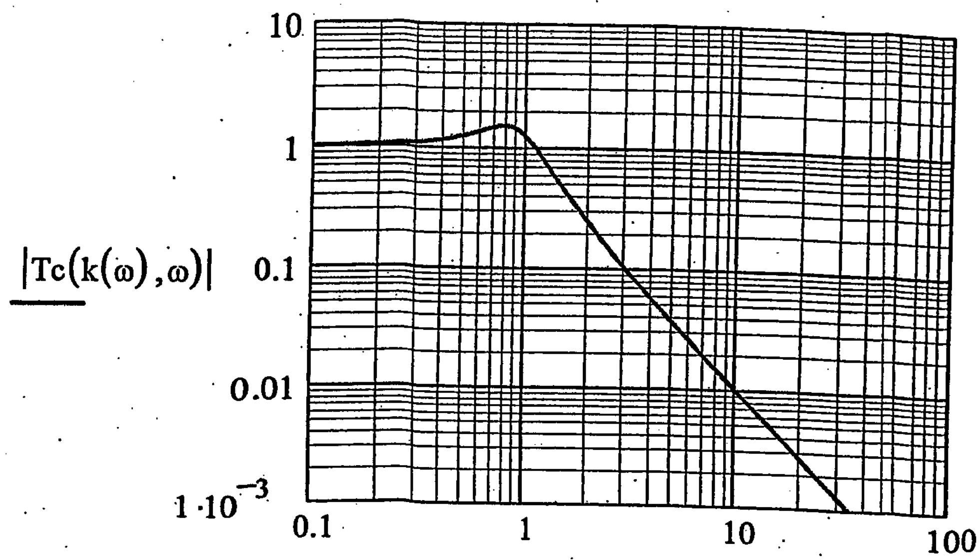


Figure I5b. Figure I5a is repeated except that the coating is absent.

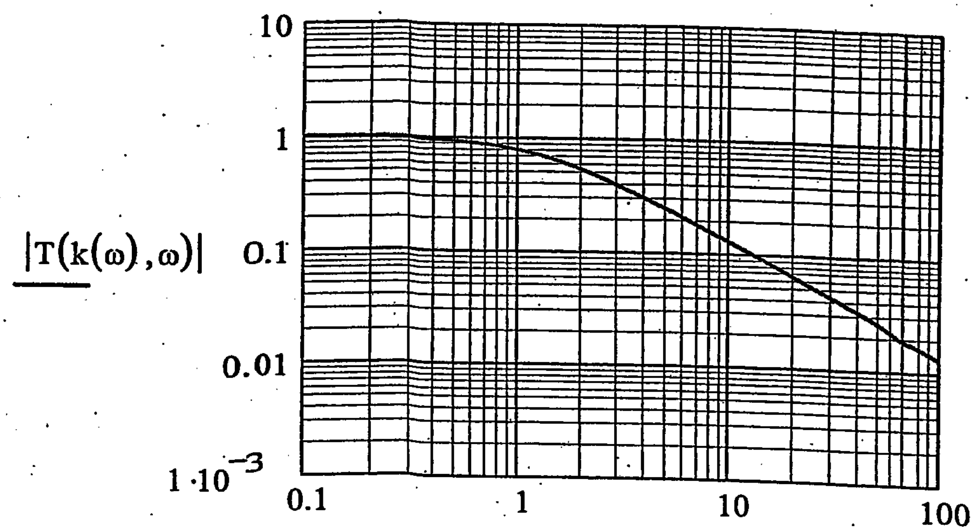
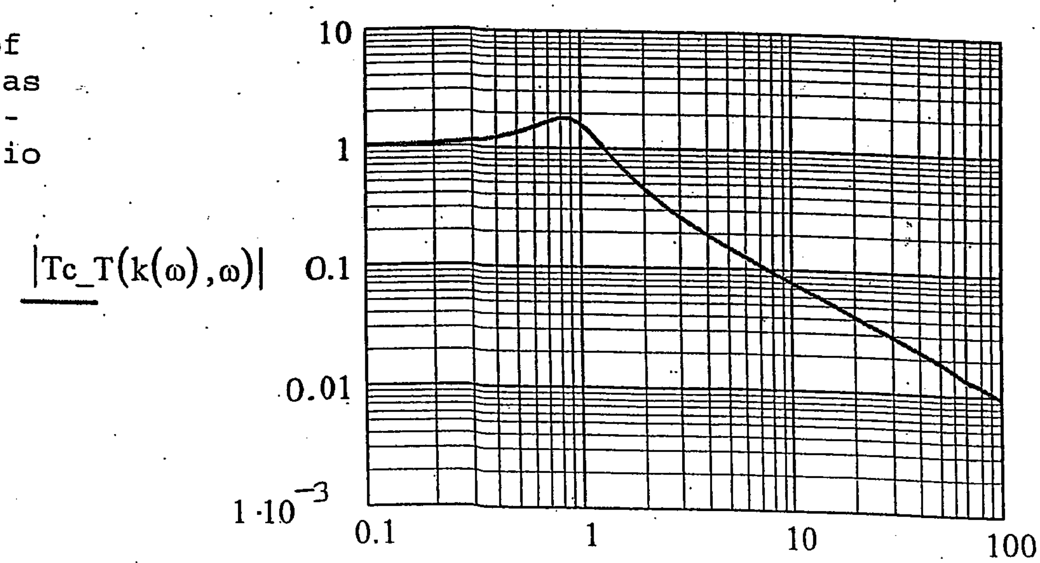


Figure I5c. The ratio of the transfer functions, as a function of the normalized frequency. The ratio is of the transfer function depicted in Figure I5a to that depicted in Figure I5b. This figure assesses the influence of the coating in the case depicted in Figure I5a.



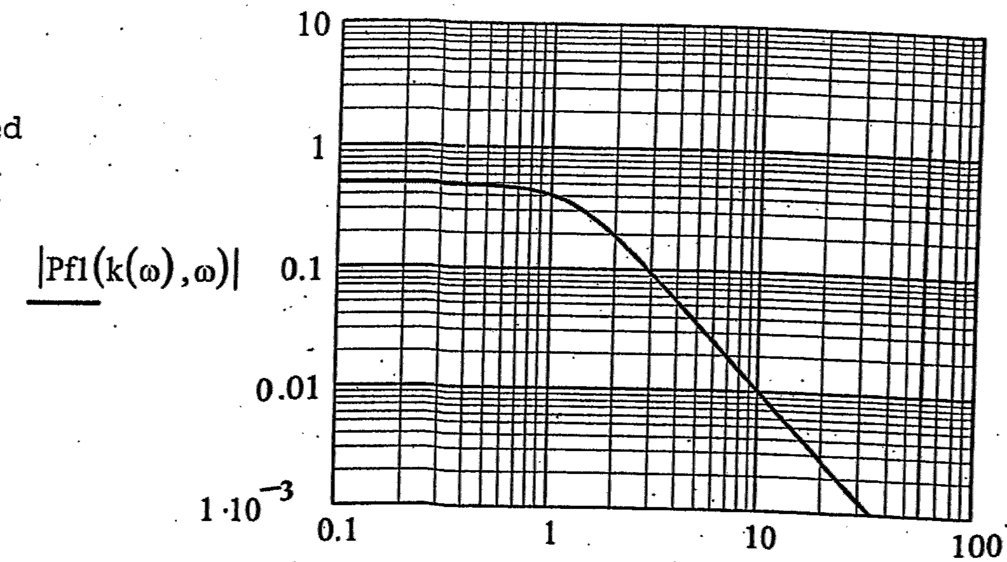


Figure I6a. The coated transfer function, as a function of the normalized frequency, under standard conditions. The transfer function of reference is that of the pressure field on the interface of the top fluid (fluid no. 1) with the coating.

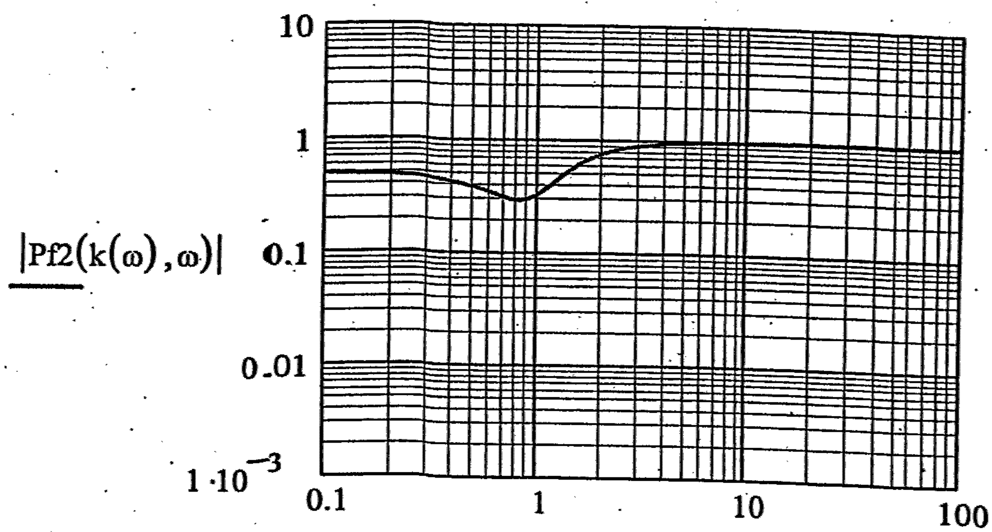


Figure I6b. The coated transfer function, as a function of the normalized frequency, under standard conditions. The transfer function of reference is the pressure field on the interface of the bottom fluid (fluid no. 2) with the equivalent plate.

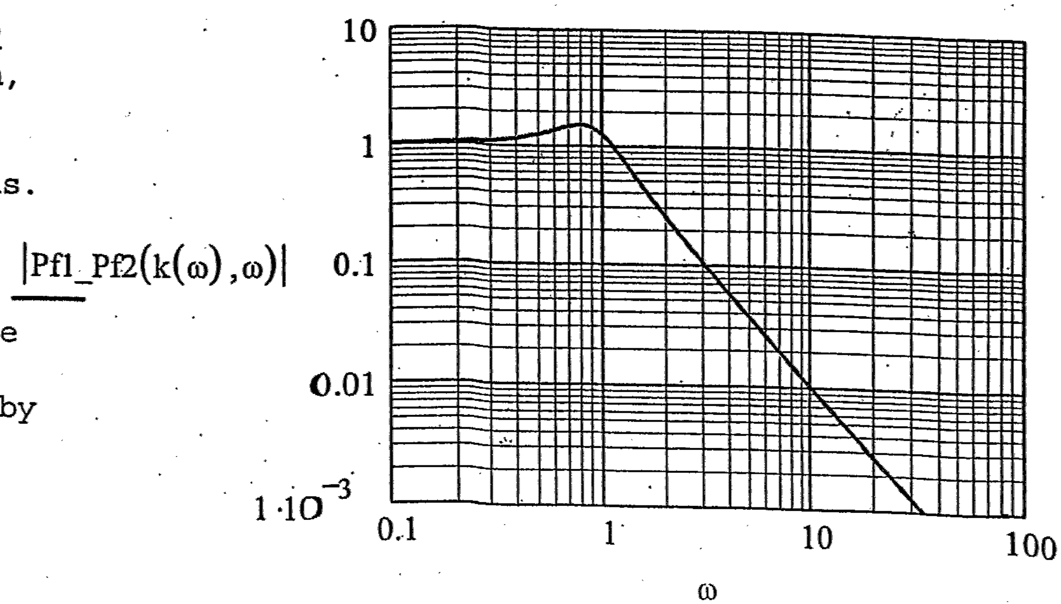


Figure I6c. The *special* coated transfer function, as a function of the normalized frequency, under standard conditions. The *special* is in reference to the normalization of the pressure *on* the interface of the top fluid (fluid no. 1) with the coating by the pressure on the interface of the bottom fluid (fluid no. 2) with the equivalent plate.

Figure I6d. Figure I6a is repeated except that the radiation is $(\pi/3)$ off beam-directed; $k = \omega \sin(\pi/3)$.

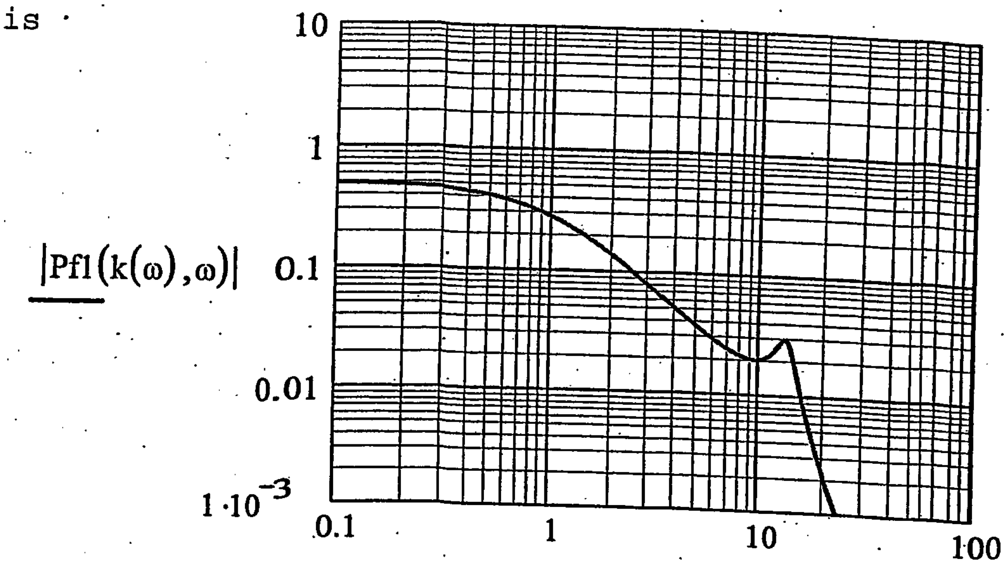


Figure I6e. Figure I6b is repeated except that the standard value of (k) is changed from zero to $k = \omega \sin(\pi/3)$.

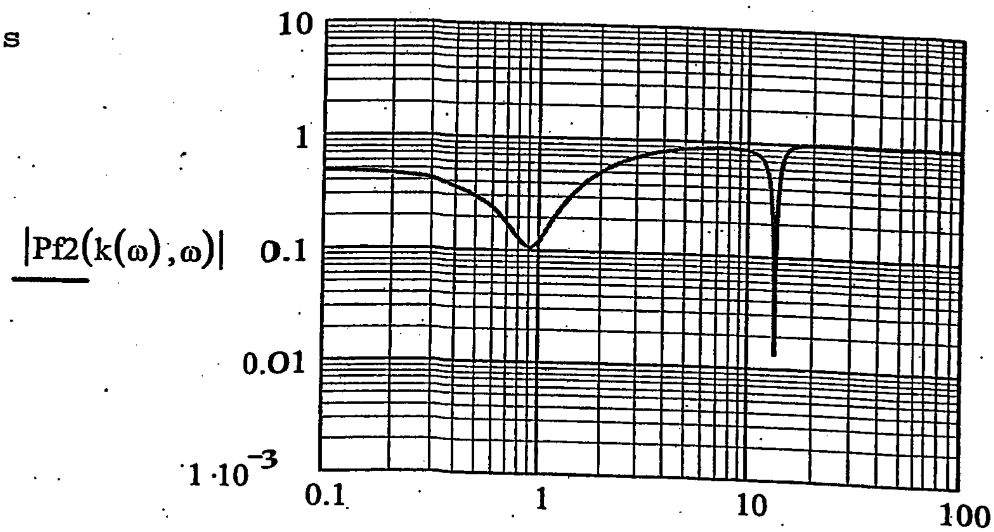


Figure I6f. Figure I6c is repeated except that the standard value of (k) is changed from zero to $k = \omega \sin(\pi/3)$.

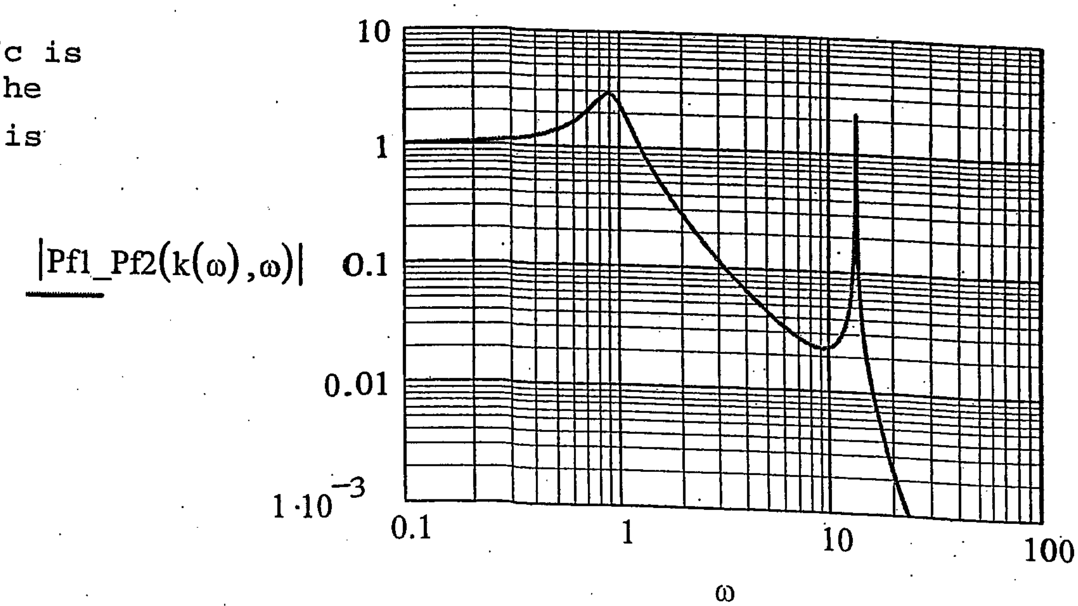


Figure I6g. Figure I6d is repeated except that the normalized critical frequency (ω_c) is changed from the standard value of ten (10) to five (5).

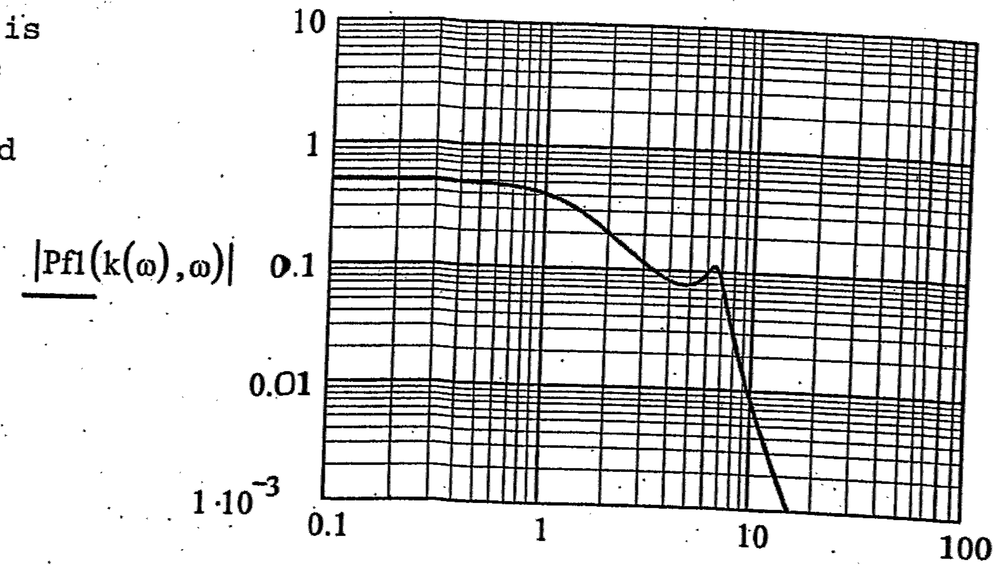


Figure I6h. Figure I6e is repeated except that the normalized critical frequency (ω_c) is changed from the standard value of ten (10) to five (5).

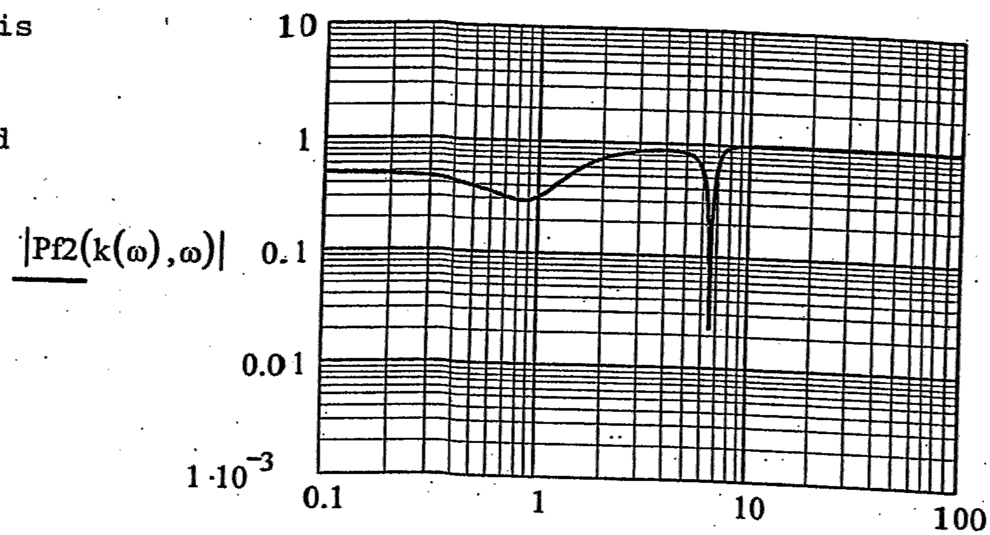
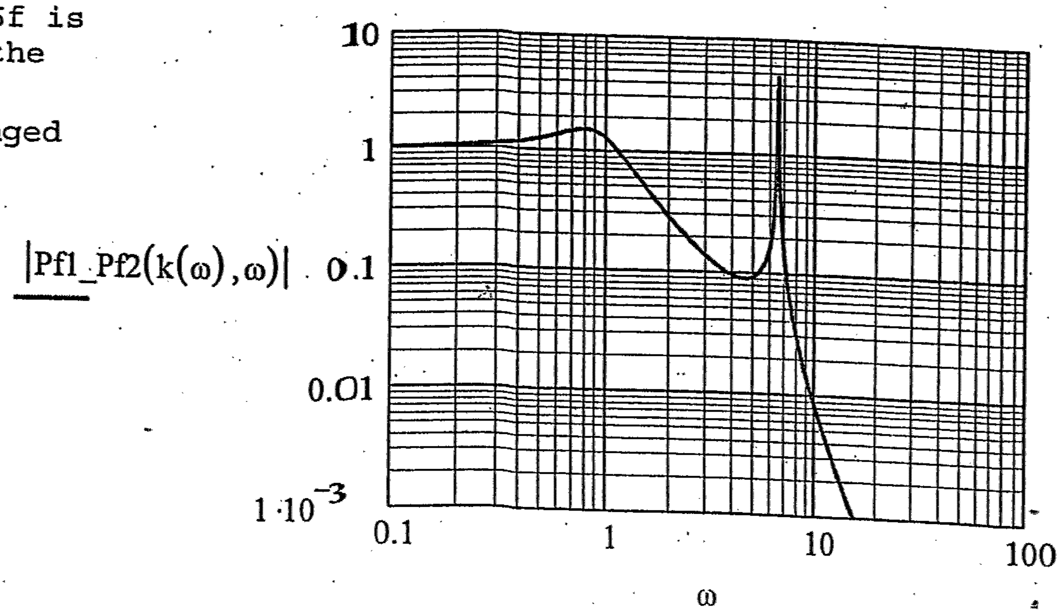


Figure I6i. Figure I6f is repeated except that the normalized critical frequency (ω_c) is changed from its standard value of ten (10) to five (5).



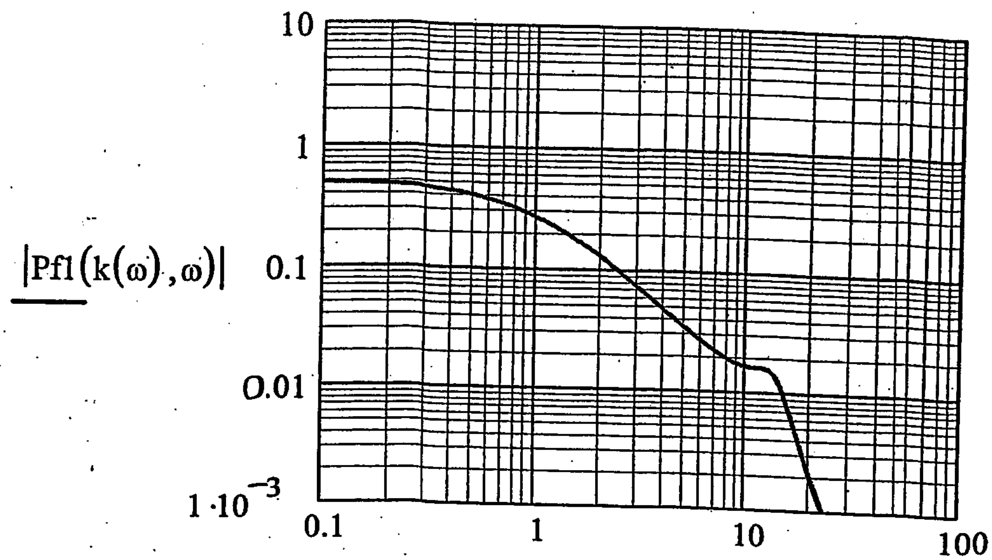


Figure I6j. Figure I6d is repeated except that the loss factors (η_m) and (η_p) in the equivalent plate are changed from their standard value of (10^{-3}) to (10^{-1}).

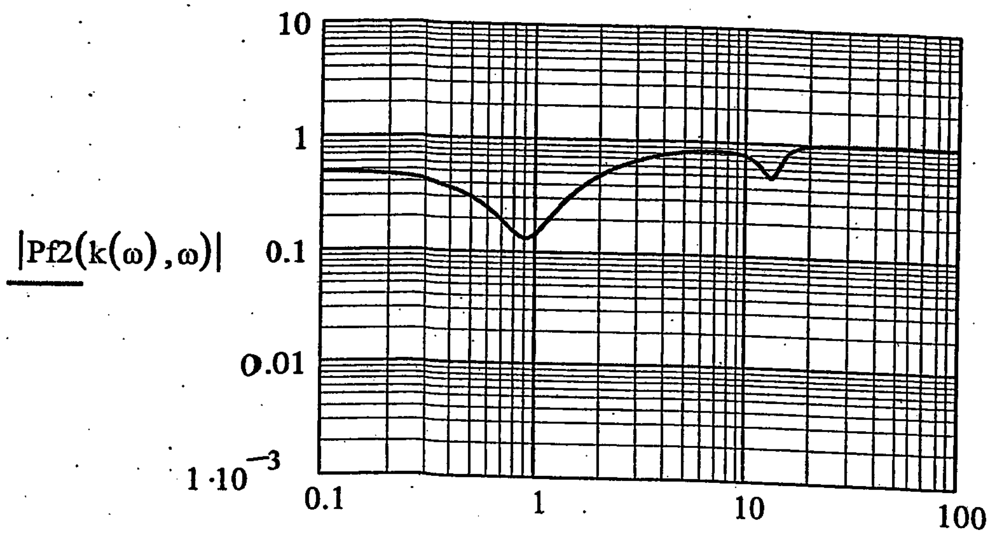


Figure I6k. Figure I6e is repeated except that the loss factors (η_m) and (η_p) in the equivalent plate are changed from their standard value of (10^{-3}) to (10^{-1}).

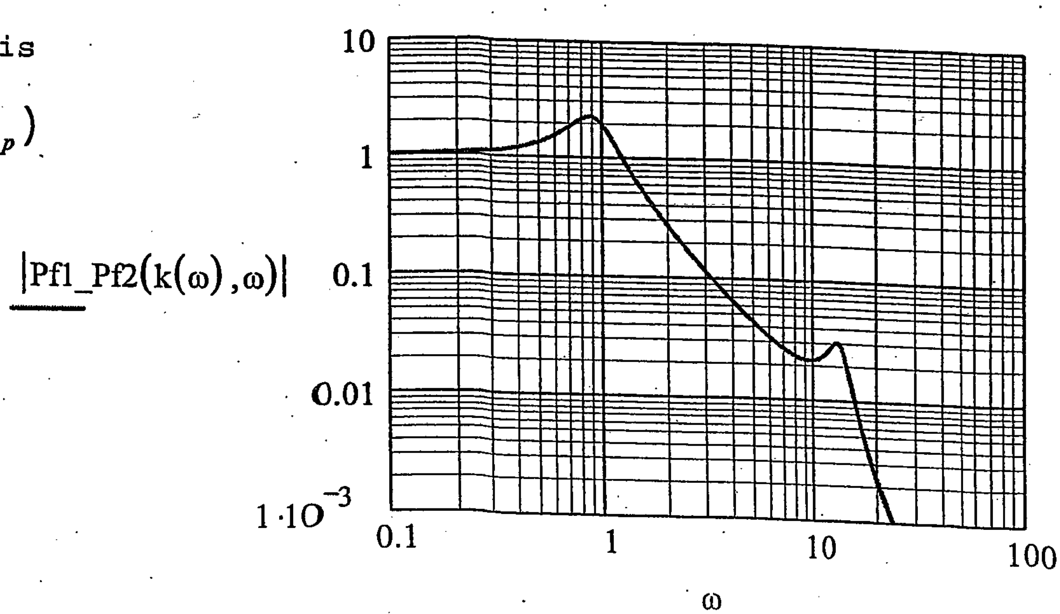


Figure I6l. Figure I6f is repeated except that the loss factors (η_m) and (η_p) in the equivalent plate are changed from their standard value of (10^{-3}) to (10^{-1}).

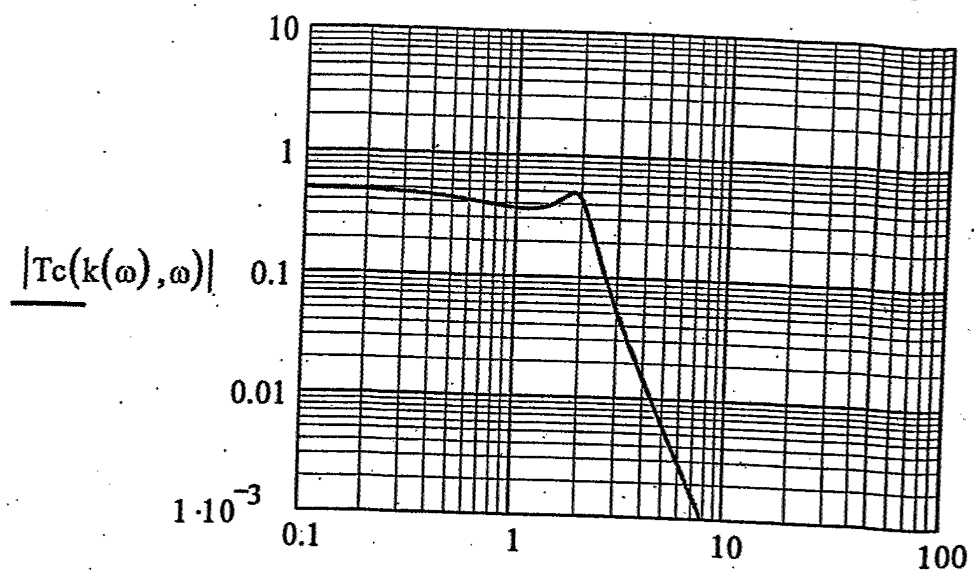


Figure I7a. The coated transfer function, as a function of the normalized frequency, for the standard conditions of the model depicted in Figure I1a except that $Z_s = Z_c$.

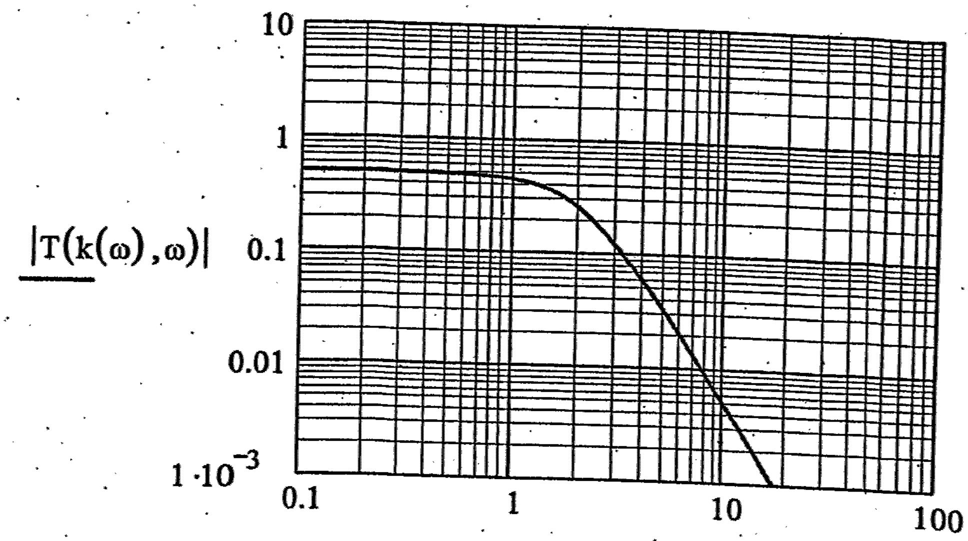


Figure I7b. Figure I7a is repeated except that the coating is removed ($|Z_c| \to \infty$, but (Z_s) remains intact).

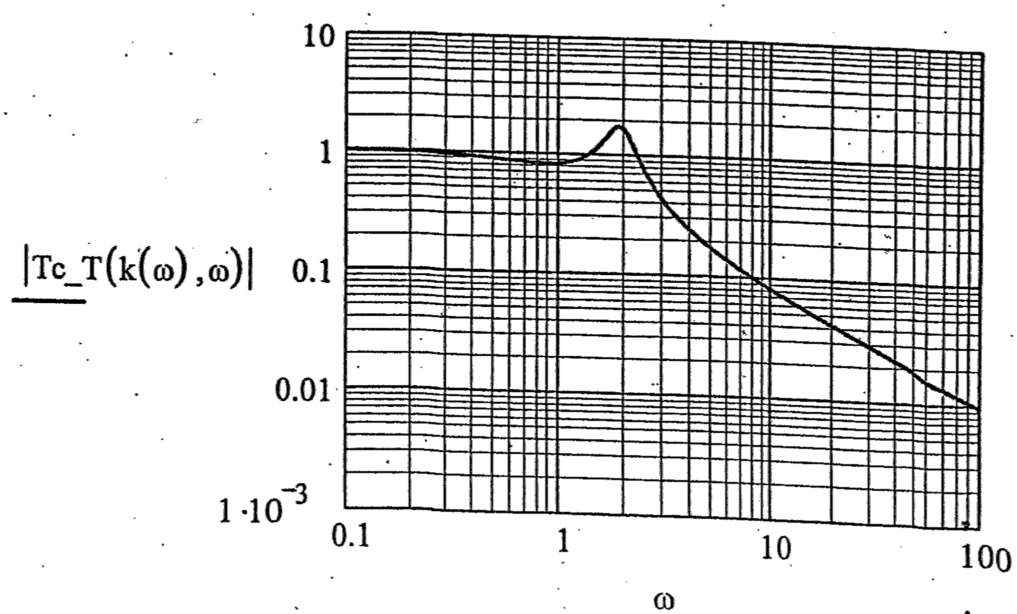


Figure I7c. The ratio of the transfer functions depicted in Figure I7a to that in Figure I7b, as a function of the normalized frequency.

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